A framework for correcting the noise-induced bias in noisy magnitude MR signals

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Outline

Introduction

- MR signals are complex numbers whose real and imaginary components are independently Gaussian distributed (1).
- The phase of the complex MRI signal is highly sensitive to many experimental factors, and as such, the magnitude of the complex MR signal is used instead.
- While the magnitude MR signal is not affected by the phase error, it is not an optimal estimate of the underlying signal intensity (1).
- Magnitude MR signals follow a Rician distribution (2,3).
- Although several correction methods have been proposed (1,3-6) to ameliorate the effects of the noise-induced bias on magnitude data, these methods do not produce corrected data that are Gaussian distributed.
- Here, we present a Signal-Transformational Framework (STF) to remove the noise-induced bias in noisy magnitude MR signals by making noisy Rician signals Gaussian-distributed.

Signal-Transformation Framework (STF) For Breaking The Noise Floor[1]

The basic idea

- A simple example illustrates the idea behind the proposed framework.
- Suppose the noisy magnitude signals are drawn from a family of Rician distributions all of which are characterized by different location parameters but with the same scale parameter (e.g., diffusion-weighted signal as a function of b-value or fMRI signal as a function of time).
- The proposed framework attempts to transform the noisy magnitude signals such that each of the noisy transformed signals may be thought of as if it were drawn from a Gaussian distribution with different mean but the same standard deviation.

[1] The work has just recently been accepted and published in the Journal of Magnetic Resonance: Vol. 197, Issue 2: Pages 108-119

- There are three stages in the proposed scheme.
- First, a data smoothing method (one, two, or higher-dimensional penalized or spline smoothing methods (7,8)) is used to obtain the average values of the noisy magnitude signals. The degree of smoothness is selected based on the method of generalized cross-validation (9).
- Second, a novel iterative method is formulated to take both an estimate of the average value of a noisy magnitude signal and an estimate of the standard deviation of the Gaussian noise to an estimate of the average value of the underlying signal intensity.
- Third, the corresponding noisy Gaussian signal of each of the noisy magnitude signals is found through a composition of the inverse cumulative probability function of a Gaussian random variable and the cumulative probability function of a Rician random variable.

- Since the first stage of the proposed scheme is readily available (7,8), our focus in this presentation will be on the later stages.

- The goal of this stage is to take in two inputs, (the average (spline-smoothed) value, denoted by $\langle m \rangle$, and the estimated Gaussian noise SD, denoted by σ_g), and return the corresponding 'average' value, η, of the underlying signal intensity.
- The fixed point formula can be shown to be:

$$\eta = \sqrt{\langle m \rangle^2 + (\xi(\eta|\sigma_g, N) - 2N)\sigma_g^2}.$$

where the scaling function ξ is given by:

$$\xi(\eta|\sigma_g, N) = 2N + \frac{\eta^2}{\sigma_g^2}(\beta_{N\,1}F_1(-1/2, N, -\eta^2/(2\sigma_g^2)))^2,$$

and $\beta_N = \sqrt{\frac{\pi}{2}} \frac{(2N-1)!!}{2^{N-1}(N-1)!}$.

- The Newton method of root-finding for finding the fixed point of the underlying signal intensity is outlined in (10).

- The goal of this stage is to transform nonCentral Chi (or Rician) distributed signals to Gaussian distributed signals.
- Mapping a nonCentral Chi random variable, m, to a Gaussian random variable, x, can be achieved by a composition of the inverse cumulative distribution function of a Gaussian random variable and the cumulative probability function of a nonCentral Chi random variable, i.e.,

$$x = P_G^{-1}(P_{\tilde{\chi}}(m|\eta, \sigma_g, N)|\eta, \sigma_g), \tag{1}$$

where the inverse cumulative distribution function of a Gaussian random variable is given by

$$P_G^{-1}(y|\eta, \sigma_g) = \eta + \sigma_g \sqrt{2} erf^{-1}(2y - 1), \tag{2}$$

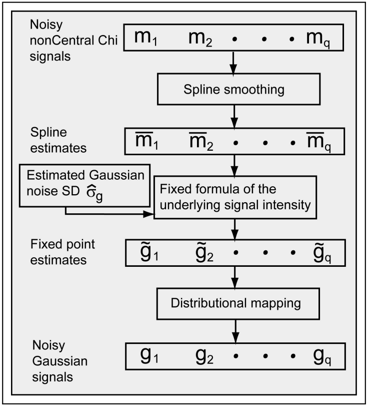
and $P_{\tilde{\chi}}(m|\eta, \sigma_g, N)$ is the nonCentral Chi cumulative distribution function, see (10). The inverse error function, $erf^{-1}(a)$, takes in a and returns b by solving the following equation: $a = \frac{2}{\sqrt{\pi}} \int_0^b \exp(-t^2) dt$

- The Newton method of root-finding for finding the fixed point of the underlying signal intensity is outlined in (10).

Signal-Transformation Framework (STF) For Breaking The Noise Floor

A schematic diagram

- In the 1st stage, the spline estimates, $\{\bar{m}_1, \cdots, \bar{m}_q\}$, are obtained from the noisy magnitude signals, $\{m_1, \cdots, m_q\}$.
- In the 2nd stage, the fixed point formula takes in each pair of data, $(\bar{m}_i, \hat{\sigma}_g)$, and turns it into a fixed point estimate $\tilde{g}_i$.
- In the 3rd stage, the distributional mapping takes in each quadruplet of data, $(\bar{m}_i, \hat{\sigma}_g, \tilde{g}_i)$, and turns it into the noisy Gaussian-distributed signal, g_i.

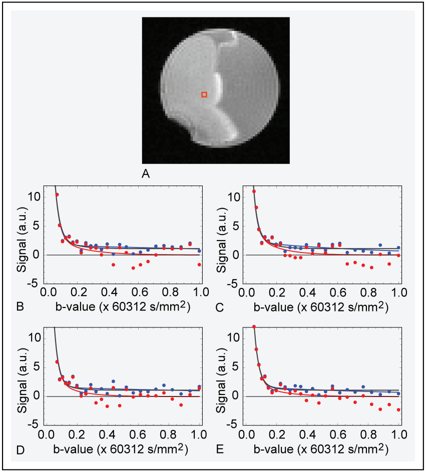


- The data set contains a series of diffusion-weighted images obtained by varying the diffusion gradient strength.
- The rat was perfusion-fixed with 4% paraformaldehyde in phosphate buffered-saline (PBS), the hippocampus was dissected and kept in fixative for more than 8 days. Prior to imaging, the sample was washed overnight in PBS.
- The imaging was performed using a 14.1T narrow-bore spectrometer where a pulsed gradient stimulated echo pulse sequence was employed.
- The imaging parameters were: $TE = 12.6ms$, $TR = 1000ms$, resolution $= (78 \times 78 \times 500)\mu\ m^3$, matrix size$=$ $(64 \times 64 \times 3)$, number of repetitions $= 4$, diffusion gradient pulse duration $(\delta) = 2ms$, and diffusion gradient separation $(\Delta) = 24.54ms$. The data set contains a total of 33 images with different diffusion gradient strengths increasing from 0 to $2935mT/m$ in steps of $91.75mT/m$.

Signal-Transformation Framework (STF) For Breaking The Noise Floor

An example based on excised rat hippocampus data set

- One diffusion weighted image is shown in Figure A. Four neighboring pixels indicated with a red square were selected for further analyses.
- The noisy magnitude signals and the noisy transformed signals of each of the pixels as a function of b-value are shown in Figures B-E as blue and red dots, respectively.
- The blue curve in each of the panels is obtained through a least squares fit of a bi-exponential function to the noisy magnitude signals. The red curve in each of the panels is obtained through a least square fit of a bi-exponential function to the noisy transformed signals produced by the STF.
- Based on the estimated Gaussian noise SD and the assumption that each of the red curves is a ground truth curve, the expected value (or the first moment) of a Rician distribution as a function of b-values can be computed and is shown in dark gray.



Signal-Transformation Framework (STF) For Breaking The Noise Floor

An example based on excised rat hippocampus data set

- The estimated parameters obtained from a least squares fit of a bi-exponential function to both the noisy magnitude signals and noisy transformed signals are shown here.

Bi-exponential fit to the noisy magnitude signals	$\hat{S}_0$ (a.u.)	$\widehat{D}_1$ ($\times 10^{-5}$ mm^2/s)	$\widehat{D}_2$ ($\times 10^{-4}$ mm^2/s)	Volume fraction associated with $\widehat{D}_1$
Fig. B	62.48	0.82	5.3	0.027
Fig. C	63.10	2.0	6.2	0.037
Fig. D	64.28	0.81	6.0	0.026
Fig. E	64.36	1.4	5.5	0.027
Bi-exponential fit to the noisy transformed signals				
Fig. B	62.6	9.0	5.5	0.060
Fig. C	63.3	10.9	6.6	0.077
Fig. D	64.4	11.3	6.2	0.056
Fig. E	64.4	9.9	5.7	0.048

Discussion

- This work (STF) can be thought of as a sequel to but independent of PIESNO (11) because the noise estimate on which the proposed framework depends may also be estimated through other techniques.
- STF and PIESNO represent our major attempt to decouple the fixed point formula of SNR (10) into two self-consistent approaches for estimating the underlying signal and the Gaussian noise SD.
- The advantage of this decoupling is substantial because the estimation of the Gaussian noise SD can be obtained from a much larger collection of samples (11).
- As a consequence, the precision of the Gaussian noise SD estimate will be significantly increased, and in turn, the precision of the underlying signal intensity estimate will also be increased.
- The combination of these stages presented here is, to the best of our knowledge, unique and novel. Moreover, the formulation of the second stage is conceptually very different from our previous approach (10).
- The basic idea of our approach is general and can be easily adapted to many MRI and non-MRI applications, e.g., the Laser Interferometric Gravitational Wave Observatory (LIGO) (12,13) and communication systems (14) , by selecting an appropriate data smoothing method that is optimal for the application-specific sampling space.

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- Software used to generate this presentation were: Beamer, LaTex, WinEdt.
- Computational and graphical software used in this work were: Mathematica, Java, Adobe Illustrator.
- STF is available in Java as an independent module. This module can be called from Matlab, IDL, and Mathematica. For further information, please visit `http://sites.google.com/site/stframework`.